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E.J. Kelly

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FOR THE COMMANDER

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SAMPLED-DATA RECEIVER DESIGN AND PERFORMANCE FOR BPSK AND MSK SPREADING MODULATIONS

E.J. KELLY
Group 41

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Abstract

Certain spread-spectrum systems make use of waveforms built of basic pulses spread by PN modulation of some kind. In this study the two most common modulation waveforms, BPSK and MSK are discussed in detail. The objective is to derive receiver structures utilizing sampled-data integration of the chips, for example, using CCD or digital correlators. A constructive design procedure is described, together with an analysis of performance in terms of SNR loss due to sampling. It is shown that sampling loss can be reduced by proper choice of the prefilter which precedes the sampling of the data.

CONTENTS

Summary

This report is concerned with certain aspects of the design of receivers for two waveforms commonly used as spreading modulations in jam-resistant communications systems. Specifically, we are interested in the reception of bursts, each many chips long, in which a PN code controls the modulation by either phase changes of 180° (BPSK) or carrier frequency shifts as in MSK. Complex signal structures can be built on these bursts. In all cases the signal parameters (including the code) are known to the receiver, and the problem of interest is to optimize its design.

It is natural to base the receiver design on the matched filter principle, insofar as this is possible with the hardware available, and one of the features of this study is a description of the form taken by a matched filter for waveforms of this type. The optimum receiver, as it might be realized at base band, forms in-phase (I) and quadrature (Q) baseband signals and passes each through a filter matched to a pulse shape characteristic of the waveform. This pulse is a rectangle, one chip long, for BPSK, and one-half cycle of a sinusoid, two chips long overall, for MSK. These outputs are then passed through delay structures which match the chip structure of the waveform. Each chip delay is tapped and weighted with the corresponding code bit and summed. The final I and Q outputs are summed and thresholded for sync detection or data demodulation.

The optimum delay structure is continuous in nature, such as a delay line. Many systems use sampled data, however, either analog or quantized in amplitude, and the sampled base-band filter outputs are then passed through a discrete delay structure, such as a CCD correlator or a digital correlator, as a practical approximation to the optimum structure.

In sampled-data systems of this kind, performance varies with "sampling phase", the relative timing of the sampling device and the chip boundaries of the actual received waveform. Pure matched filter performance is attained only for certain values of the sampling phase, and on the average a certain loss in performance occurs, usually expressed as a "sampling loss" in effective signal-to-noice ratio (SNR).

The sampling loss is the main concern of this study, and it is evaluated as a function of sampling rate (number of samples per chip) for both waveforms, using the baseband filters matched to the appropriate pulse, as described above. However, it is also shown that this sampling loss behavior

can be changed, often in a desirable way, by modifying the baseband filters, and several filter designs are presented, based on different criteria for acceptable sampling loss behavior.

The reductions in sampling loss are not great, but it is shown that the sampling loss variation with sampling phase can be sharply reduced for BPSK, even with one sample per chip, and that sampling losses with MSK sampled once per chip can be comparable to losses with BPSK sampled at twice the rate. These conclusions should be useful in receiver design, even though they deal with only one aspect of that design, since sampling rates (and chipping rates) are usually pushed as high as the technology will allow.

In addition to these details, a number of mathematical properties of MSK are discussed, including the desirability of using one of the two tones as a reference for demodulation to baseband.

I. INTRODUCTION

Many communications systems make use of waveforms built entirely of basic pulses, combined in ways to achieve the fundamental requirements of synchronization and information transmission. Synchronization implies accurate timing of the arrival of a reference point in the waveform, in support of data demodulation and, in some cases, ranging. If data modulation takes the form of pulse position modulation, then the entire task of the receiver reduces, in a sense, to the detection and timing of the arrival of the individual basic pulses of the waveform.

One approach to waveform design is to use intense, very short pulses, the other common method is to increase the bandwidth of a pulse of unmodulated carrier by modulating it. Some form of phase modulation is often preferred because of the convenience of dealing with constant-envelope pulses.

Binary phase modulation schemes have a natural advantage in that the waveform itself, as well as the matched filter or correlator included in the preferred implementation of the receiver, can be neatly specified by means of a "code", or sequence of bits. In the applications of this approach to waveform design, the binary code is used only to spread the pulse spectrum and not to convey information, since the code is already known to the receiver. Information could be transferred by transmitting one of a set of codes, but this forces the receiver to implement parallel channels, each designed to detect one of the codes, and hence each channel is basically similar to a fixed-code receiver, with analogous requirements on the pulses.

Two of the simplest modulation schemes which translate these ideas and constraints to practice are binary phase shift keying (BPSK) and minimum-shift keying (MSK). In each case the pulse is divided into a number of equal segments, called chips, one for each code bit. In BPSK, the carrier phase takes one of two values in each chip, according to the corresponding code bit, while in MSK the instantaneous frequency takes one of two values, in analogous fashion. In BPSK, the two values of phase differ by 180°, while in MSK the difference of the two frequencies accumulates a phase difference of 180° over the duration of one chip.

This report is a study of a class of receiver structures designed to detect and time basic pulses, spread by either BPSK or MSK. The chief characteristic of these receivers is their use of sampled-data correlators for the implementation of a portion of the processing, and a main objective of the study is the optimization of the remaining portion of the receiver in order to minimize the inherent loss in performance due to sampling. The emphasis is on detection performance, but the results obtained are also useful in assessing timing accuracy as well. In addition, some insight is gained into system trade-off issues, relating sampling rate and correlator structure to performance.

II. THE MATCHED FILTER

In this section we review the simple theory of the matched filter. This will serve to establish some basic results in the general case, avoiding repetition for specific waveforms, and it will also introduce some conventions of notation.

The received waveform is represented as a modulated carrier:

$$R(t)\cos[\omega_o t + \Phi(t)] \quad ,$$

and the function we deal with is the complex modulation

$$Z(t) = R(t)e^{i\Phi(t)} \quad .$$

This representation is not unique, of course, since the same waveform can be expressed in terms of a different carrier frequency, as follows:

$$R(t)\cos[\omega_o' t + \Phi'(t)] \quad ,$$

where

$$\Phi'(t) = \Phi(t) + (\omega_o - \omega'_o)t \quad .$$

This is a simple point, of course, but in the discussion of MSK it proves useful to consider two such representations, with slightly different carrier frequencies.

The real and imaginary components of $Z(t)$ are recovered, as in-phase (I) and quadrature (Q) signals, by beating the received signal to baseband with a local oscillator at the chosen carrier frequency (together with a 90-degree phase shifter to provide the quadrature reference).

The complex modulation is represented as a sum of signal plus noise:

$$Z(t) = Ae^{i\beta}\, S(t-\tau) + N(t) \quad .$$

Here, A and β represent amplitude and carrier phase, while $S(t)$ describes the basic signal waveform, usually normalized to unit amplitude. The parameter τ represents the actual signal arrival time, and $N(t)$ describes the complex noise. This noise is assumed to be white with the properties

$$E\, N(t) = E\, N(t)\, N(t') = 0 \quad ,$$

$$E\, N^*(t)\, N(t') = 2N_o\, \delta(t'-t).$$

The real and imaginary parts of N(t) are uncorrelated, and N_o is the single-sided power spectral density. E stands for expectation value, or ensemble average.

The output of the matched filter to an input Z(t) is taken to be

$$W(t) = \int S^*(\sigma) Z(\sigma + t) d\sigma \quad .$$

For convenience, we have expressed the matched filter output as a complex correlation with the signal waveform, rather than a convolution with the corresponding filter impulse response. Also, no attempt is made to account for realizability delays. We also use the convention that all integrals run from $-\infty$ to $+\infty$, since in every case the integrals contain signals of finite duration as factors.

We are concerned here with incoherent reception, hence detection and timing of signals are based upon the magnitude, $|W(t)|$, obtained as the root-sum-square of the I and Q components of the filter output. If the same received waveform is represented in terms of another carrier frequency, ω'_o, the modulation will be

$$Z'(t) = e^{i(\omega_o - \omega'_o)t}\, Z(t) \quad ,$$

and the signal component will be

$$S'(t) = e^{i(\omega_o - \omega'_o)t}\, S(t) \quad .$$

The corresponding matched filter output will be

$$W'(t) = \int S'^*(\sigma)\, Z'(\sigma + t) d\sigma = e^{i(\omega_o - \omega'_o)t}\, W(t) \quad .$$

Since W'(t) has the same magnitude as W(t), there will be no difference in performance of the filters corresponding to the two choices of carrier, and again this will be useful in analysis of receivers for MSK.

In the absence of noise, the output would be due to signal only:

$$W(t) = Ae^{i\beta}\int S^*(\sigma)S(\sigma + t - \tau)d\sigma$$

$$= Ae^{i\beta}\, C(t - \tau) \quad ,$$

where C(t) is the signal autocorrelation function:

$$C(t) = \int S^*(\sigma)S(\sigma + t)d\sigma \quad .$$

This output has its peak value at $t=\tau$:

$$W(\tau) = Ae^{i\beta}C(0) \quad .$$

For noise alone, the output is

$$W(t) = \int S^*(\sigma)N(\sigma + t)d\sigma \quad ,$$

a random process with autocorrelation function

$$EW^*(t)\ W(t') = 2N_o \iint S(\sigma)S^*(\sigma')\delta(\sigma + t - \sigma' - t')d\sigma\, d\sigma'$$

$$= 2N_o C(t'-t) \quad .$$

The output signal-to-noise ratio (SNR) is defined as the ratio of $|W(t)|^2$, for signal only, to $E|W(t)|^2$, for noise alone. The peak SNR, which occurs when $t=\tau$, is

$$\mathrm{SNR} = \frac{A^2C(0)}{2N_o} = E_s/N_o,$$

where E_s, the total signal energy, is

$$E_s = \frac{A^2}{2}\, C(0) = \frac{A^2}{2}\int |S(\sigma)|^2\, d\sigma \, .$$

The signals of interest in this report are of constant envelope: $|S(t)| = 1$, and of finite duration, say T, so that $E_s = (1/2)\ A^2\ T$.

In later sections we will be discussing approximate implementations of matched filters, and their performance will be characterized by the loss in peak output SNR, relative to E_s/N_o .

III. THE BPSK SIGNAL AND MATCHED FILTER

The BPSK signal can be described by the modulation function

$$S(t) = \sum_n a_n P_o(t-n\Delta) \quad .$$

The a_n are binary variables, assuming values ± 1, for n in the range $1 \leq n \leq L$. They represent some pseudorandom code, whose detailed properties are not of interest here, except for sidelobes, as discussed below. We use the convention that $a_n = 0$ for $n < 0$ and $n > L$, so that all sums are unrestricted in extent. The chip duration is Δ, and the basic pulse, P_o is defined by

$$P_o(t) \doteq \begin{cases} 1 \; ; & -\frac{\Delta}{2} \leq t \leq \frac{\Delta}{2} \\ 0 \; ; & \text{otherwise} \end{cases} \quad .$$

It is unnecessary to make any assumption about the relation between carrier frequency and chip duration, since the analysis uses only the modulation. In practical schemes for generation of BPSK it may be useful to work at a carrier which is a multiple of the chipping rate, but this relation is lost when the carrier is changed, either before transmission or in the receiver as part of the demodulation process.

In terms of ρ_o, the normalized autocorrelation function of P_o:

$$\rho_o(t) = \frac{1}{\Delta} \int P_o(\sigma)\, P_o(\sigma + t)d\sigma$$

$$= \begin{cases} 1 - \frac{|t|}{\Delta} & ; -\Delta \leq t \leq \Delta \\ 0 & ; \text{ otherwise} \end{cases} \quad ,$$

the BPSK signal autocorrelation function is

$$C(t) = \sum_{n,m} a_n a_m \int P_o(\sigma - n\,\Delta) P_o(\sigma + t - m\,\Delta) d\sigma$$

$$= \Delta \sum_{\ell} C_{\ell}\, \rho_o(t - \ell\Delta) \quad .$$

We have introduced the code autocorrelation sequence

$$C_{\ell} \equiv \sum_{n} a_n a_{n+\ell} \quad .$$

Note that $C_o = L$, $C_{-\ell} = C_{\ell}$ and that, for $\ell \geq 0$,

$$C_{\ell} = \sum_{n=1}^{L-\ell} a_n a_{n+\ell} \quad ,$$

so that $C_{\ell} = 0$ for $|\ell| \geq L$. For "good" code sequences, the sidelobe values (C_{ℓ} for $\ell \neq 0$) will be small compared to L. In any case,

$$C(0) = \Delta\, C_o\, \rho_o(0) = L\,\Delta \ ,$$

which equals to signal duration, hence

$$E_s = \frac{1}{2} A^2 L\,\Delta \quad .$$

The matched filter processor takes a simple form, since we may write

$$W(t) = \sum_{n} a_n \int P_o(\sigma - n\,\Delta)\; Z(\sigma + t) d\sigma$$

$$= \sum_{n} a_n\; Z_1(t + n\,\Delta) \ ,$$

where

$$Z_1(t) \equiv \int P_o(\sigma)\; Z(\sigma + t) d\sigma \ .$$

Now Z_1 (t) can be obtained by passing Z(t) through a filter, in fact a filter matched to the pulse P_o, again with the realizability delay ignored. It is immaterial whether this filtering is accomplished at bandpass, with the signal on a carrier, or as a pair of identical filters (since P_o is real) operating on the baseband I and Q components of Z(t). The desired output, W(t), is obtained from the prefiltered waveform, $Z_1(t)$, by passing the latter through a tapped delay line structure. Each segment of this line introduces delay Δ, and the tapped outputs are weighted by the code weights, a_n, and then summed. Again, our formula expresses the output as a correlation instead of a convolutions, and the realizability delay is ignored. The delay line structure can be implemented on a carrier or (since the a_n are real) as a pair of identical structures operating on the prefiltered baseband components of $Z_1(t)$.

The output SNR of the matched filter, as a function time, will be

$$\begin{aligned}
SNR(t) &= \frac{A^2 |C(t-\tau)|^2}{2N_o C(0)} \\
&= \frac{E_s}{N_o} \left[\frac{C(t-\tau)}{C(0)}\right]^2 \\
&= \frac{E_s}{N_o} \left[\frac{1}{L} \sum_{\ell} C_{\ell}\ \rho_o(t-\tau-\ell\Delta)\right]^2 .
\end{aligned}$$

The expression inside the bracket takes the value unity at $t = \tau$, the signal arrival time, and the values C_ℓ/L at times $t = \tau + \ell\Delta$. Moreover, this sum varies linearly between these values at intermediate times, as is easily seen by substituting for $\rho_o(t)$. Since the filter produces a continuous output in time, the peak value, E_s/N_o, of SNR(t) is always attained.

IV. THE MSK SIGNAL AND MATCHED FILTER

We introduce the MSK modulation as pure frequency modulation, where within each chip the radian frequency is either increased or decreased by a fixed amount, ν, according to the value of a code bit. The value of ν is related to the chipping rate so that the modulation produces a phase change of $\pm$ 90 degrees over one chip. We put

$$S(t) = \begin{cases} e^{i\Phi(t)} & ; \quad 0 \leq t \leq L\,\Delta \\ 0 & ; \quad \text{otherwise} \end{cases}$$

where the initial phase, $\Phi(0)$, is zero, and the instantaneous frequency is

$$\dot{\Phi}(t) = b_n\,\nu\ , \quad \text{for } (n-1)\,\Delta \leq t < n\,\Delta\ .$$

The b_n are binary variables representing a code sequence, and n runs from 1 through L. The relation between modulation frequency and chip length is simply

$$\nu\,\Delta = \pi/2\ .$$

The phase, $\Phi(t)$, is a continuous function of time, which takes on the values $\Phi(\Delta) = b_1\pi/2$, $\Phi(2\Delta) = (b_1+b_2)\pi/2$, and generally

$$\Phi(n\Delta) = (b_1 + \ldots + b_n)\pi/2\quad ,$$

at the chip transition times. At an intermediate time the phase varies linearly:

$$\Phi(t) = \Phi(n\Delta) + b_{n+1}\,\nu(t-n\Delta)\quad ,$$

for $\quad n\,\Delta \leq t \leq (n+1)\Delta\ .$

For a binary variable, b, we have the obvious identity

$$e^{i\,b\,\theta} = \cos\theta + i\,b\,\sin\theta \quad ,$$

and in particular

$$e^{i\,b\,\pi/2} = i\,b \quad .$$

Therefore, the modulation function can be written in the form

$$S(t) = e^{i\,(b_1 + \,\ldots\, + b_n)(\pi/2) \,+\, i\,b_{n+1}\nu(t\,-\,n\Delta)}$$

$$= i^n b_1 \ldots b_n \,[\cos\,\nu\,(t - n\Delta) + i\,b_{n+1}\sin\,\nu(t - n\Delta)],$$

for t in the range $[n\,\Delta,\,(n+1)\Delta]$. This formula holds for t in the range $[0,\,\Delta]$ if we interpret the product $b_1 \ldots b_n$ to be unity for $n = 0$. We define the new sequences of binary variables:

$$a_o \equiv 1$$

$$a_n \equiv b_1 \ldots b_n \quad , \quad 1 \leq n \leq L \quad ,$$

and observe that

$$S(t) = i^n a_n \cos\,\nu\,(t-n\Delta) + i^{n+1}\,a_{n+1}\,\cos\,\nu\,[t - (n+1)\Delta] \quad ,$$

for t in the range $[n\,\Delta,\,(n+1)\Delta]$ and $0 \leq n < L$. We have used the relation of ν to Δ to equate $\sin\,\nu\,t$ to $\cos\,\nu(t-\Delta)$.

If we define the "MSK pulse", $P(t)$, by

$$P(t) = \begin{cases} \cos\,\nu t & ; \quad -\Delta \leq t \leq \Delta \\ 0 & ; \quad \text{otherwise} \end{cases} \quad ,$$

then

$$S(t) = i^n a_n P(t-n\Delta) + i^{n+1} a_{n+1} P[t-(n+1)\Delta] \quad ,$$

for t in $[n\Delta,(n + 1)\Delta]$. But for t in this range, the sum can be extended:

$$S(t) = \sum_{n=o}^{L} i^n a_n P(t-n\Delta) \quad ,$$

since the other terms vanish by the definition of P(t). Obviously, this sum is a correct expression for S(t) for any t in whole range $[0,L\Delta]$, and it serves to represent the MSK modulation as a sum of overlapping pulses with complex weights of a particular kind. It should be noted that the code sequence used in this representation is not the original sequence, which is easily recovered from the identity

$$b_n = a_n a_{n-1} \quad , \qquad (1 \leq n \leq L) \quad .$$

This representation is so useful, both for analysis and as a model for the actual generation of MSK, that we redefine S(t) to be the value of this sum for all t, even outside the interval $[0,L\Delta]$. This means that we allow the waveform to depart from the constant-envelope form in the end intervals $[-\Delta, 0]$ and $[L\Delta,(L + 1)\Delta]$, where a single cosine-term spills over. We take the a_n-sequence to be the basic code, and make one final change, by dropping the term n = o, so that

$$S(t) \equiv \sum_{n=1}^{L} i^n a_n P(t-n\Delta) \quad ,$$

which vanishes outside the range $[0,(L + 1)\Delta]$. The envelope is unity within $[\Delta, L\Delta]$, and it is easy to see that

$$\int |S(t)|^2 dt = L\Delta \quad .$$

By making this last change we gain a consistency with the BPSK formulation, and we also use the convention that $a_n = 0$ unless $1 \leq n \leq L$, so that sums can be unrestricted in range.

If the carrier radian frequency is ω_o, then the MSK waveform has an instantaneous frequency of either $\omega_o + \nu$ or $\omega_o - \nu$. Looked at another way, the actual waveform presents one of two frequencies, which differ by half the chipping rate, and our representations of the modulation refer to a carrier which is the average of these two frequencies. Using the freedom to change the carrier frequency to represent the same real waveform, which was discussed in Section II, we can take one of the two MSK frequencies as carrier, and study the resulting modulation function. It proves useful to take the upper frequency, $\omega_o + \nu$, as the alternative carrier, for which the modulation function will be called $Z'(t)$. The corresponding signal modulation is

$$S'(t) = e^{-i\nu t} S(t)$$

since, obviously

$$S'(t) e^{i(\omega_o + \nu)t} = S(t) e^{i\omega_o t} \quad .$$

Substituting, we find

$$S'(t) = e^{-i\nu t} \sum_n i^n a_n P(t-n\Delta)$$

$$= \sum_n a_n e^{-i\nu(t-n\Delta)} P(t-n\Delta) \quad ,$$

or

$$S'(t) = \sum_n a_n P'(t-n\Delta) \quad ,$$

where

$$P'(t) \equiv e^{-i\nu t} P(t)$$

$$= \begin{cases} \frac{1}{2}(1 + e^{-2i\nu t}) & ; \quad -\Delta \le t \le \Delta \\ 0 & ; \quad \text{otherwise} \quad . \end{cases}$$

It is easy to see how MSK results from this modulation. At any time, only two sucessive terms contribute to S'(t), and if the corresponding code bits are equal, the upper frequency results, since

$$P'(t) + P'(t-\Delta) = 1.$$

If the bits are different, the lower frequency results because

$$P'(t) - P'(t-\Delta) = e^{-2i\nu t},$$

and this modulation converts the carrier, $\omega_o + \nu$, into the lower frequency, $\omega_o - \nu$.

The representation using S'(t) also explains a common method of generation of MSK. It can be directly verified that

$$P'(t) = \int H(s)\, P_o(t-s)\, ds \quad ,$$

where

$$H(s) \equiv \nu P_o(s) e^{-2i\nu s} \quad .$$

The proof follows easily from the expression

$$P'(t) = \nu \int_{-\Delta/2}^{\Delta/2} e^{-2i\nu s}\, P_o(t-s)ds = \nu e^{-2i\nu t} \int_{t-\frac{\Delta}{2}}^{t+\frac{\Delta}{2}} e^{2i\nu\sigma}\, P_o(\sigma)d\sigma \quad .$$

Thus P'(t) is the response of a filter to the basic pulse $P_o(t)$; the impulse response of the filter is H(s) (excluding the realizability delay), which represents a filter matched to a burst, one chip in duration, of the lower frequency.

Using this fact, we can write

$$S'(t) = \int H(s) \left\{ \sum_n a_n P_o(t-n\Delta-s) \right\} ds \ ,$$

which shows that $S'(t)$ can be formed by generating a BPSK waveform, using the upper frequency and code sequence a_n, and passing this waveform through the filter described by the modulation function $H(s)$.

Both representations of MSK are useful, and they suggest different realizations of the matched filter processor. In either case, the filter output SNR is determined by the magnitude of the signal autocorrelation function. Using the S-representation (i.e. the carrier ω_o), we have

$$C(t) = \int S^*(\sigma)S(\sigma + t)d\sigma$$

$$= \sum_{n,m} i^{m-n} a_n a_m \int P(\sigma - n\Delta)P(\sigma + t - m\Delta)d\sigma$$

$$= \Delta \sum_{\ell} i^{\ell} C_{\ell}\rho(t - \ell\Delta) \ .$$

In this formula, C_ℓ is the code autocorrelation, as before, and $\rho(t)$ is the normalized autocorrelation function of the MSK pulse:

$$\rho(t) = \frac{1}{\Delta}\int P(\sigma)P(\sigma + +)d\sigma$$

$$= \begin{cases} (1 - \frac{|t|}{2\Delta})\cos \nu t + \frac{1}{\pi} \sin(\nu|t|) & ; \quad -2\Delta \le t \le 2\Delta \\ 0 & ; \quad \text{otherwise.} \end{cases}$$

Note that this function is non-zero for a time four chips in duration. Thus, in general, many terms contribute to the value of C(t) at any given time, but for $t = 0$,

$$C(0) = \Delta\, C_o \rho(0) = L\, \Delta \quad ,$$

since $\rho(\ell\Delta)$ vanishes for $|\ell| > 1$, and the terms $\ell = 1$ and $\ell = -1$ cancel (C_ℓ and $\rho(\ell\Delta)$ are even functions of ℓ) . Had we used the S'-representation (carrier $\omega_o + \nu$), we would have found the autocorrelation function

$$C'(t) = e^{-i\nu t} C(t) \quad ,$$

but in either case the output SNR is

$$SNR(t) = \frac{E_s}{N_o} \left| \frac{C(t - \tau)}{C(0)} \right|^2$$

for a signal with arrival time τ .

The matched filter structure follows exactly as in the BPSK case. Starting with the S-representation, we define the prefiltered modulation function $Z_1(t)$:

$$Z_1(t) \equiv \int P(\sigma) Z(\sigma + t) d\sigma \quad ,$$

and find that

$$W(t) = \int S^*(\sigma) Z(\sigma + t) d\sigma$$

$$= \sum_n a_n i^{-n}\, Z_1(t + n\,\Delta) \quad .$$

Note that $Z_1(t)$ is formed by passing $Z_1(t)$ through a filter matched to P(t), considering ω_o to be the carrier. This can be realized as a bandpass filter or, alternatively, the filtering could be performed on baseband signals obtained using the average frequency as the reference. A problem arises in the next step, the tapped delay line structure, since the weights, $a_n i^{-n}$, are complex. To implement this literally at baseband requires a pair of tapped delay lines, in which the even-numbered taps from one line are weighted and combined with the weighted, odd-numbered taps of the other line to form one output. The remaining taps of the two lines are used to form the other output, representing the real and imaginary parts of W(t), according to the equations

$$\mathrm{Re}\{W(t)\} = \sum_n (-1)^n a_{2n} X_1(t + 2n\,\Delta) + \sum_n (-1)^n a_{2n+1} Y_1[t + (2n+1)\Delta]$$

$$\mathrm{Im}\{W(t)\} = \sum_n (-1)^n a_{2n} Y_1(t + 2_n\,\Delta) - \sum_n (-1)^n a_{2n+1} X_1[t + (2n+1)\Delta],$$

where X_1 and Y_1 are the I and Q components of Z_1:

$$Z_1(t) = X_1(t) + i\,Y_1(t) \quad .$$

The same effect has been very closely approximated in a SAW structure, operating as a bandpass filter and delay structure, in which each element of delay is decreased by one-fourth the local carrier wavelength to effect the 90-degree phase shifts required by the successive factors i^{-n} .

Another matched filter structure results from using the S-representation, where the received modulation is Z'(t) and the upper MSK tone is considered to be the carrier. We immediately find that

$$W'(t) = \int S'^*(\sigma)\, Z'(\sigma + t)\, d\sigma$$

$$= \sum_n a_n Z_1'(t + n\,\Delta) \quad ,$$

where

$$Z_1'(t) \equiv \int P'^*(\sigma)\, Z'(\sigma + t)d\sigma \quad .$$

The delay line structure is now identical to that for BPSK, with real weights, a_n, and the complexity has been shifted to the prefiltering which produces $Z'_1(t)$. Since $P'^*(\sigma) = P'(-\sigma)$, we have

$$Z_1'(t) \quad = \int P'(\sigma)Z'(t-\sigma)d\sigma \quad ,$$

hence $Z_1'(t)$ is obtained by passing $Z'(t)$ through a filter matched to $P'(t)$. The impulse response of this filter is a sum of the two MSK frequencies.

It is easier, however, to represent this filter as a cascade of two filters, using our previous result

$$P'(t) \quad = \int H(s)P_o(t - s)ds \quad .$$

Thus $Z'(t)$ can be passed first through the filter described by $H(t)$, and then through the filter matched to $P_o(t)$, (or vise versa) according to

$$Z_1'(t) \quad = \int\int P_o(s)\, H(\sigma)Z'(t - s - \sigma)\, d\sigma\, ds \quad .$$

All this filtering can be accomplished on a carrier, using appropriate bandpass filters.

Since the real signal described by modulation $Z_1'(t)$ and carrier $\omega_o + \nu$ is identical to the signal described by $Z_1(t)$ and carrier ω_o, it is obvious that the filtering operations described by $P(t)$ in the S-representation and $P'(t)$ in the S-representation are identical if carried out at bandpass. Having performed this bandpass filtering, we are then led to the requirement of complex weights if we use ω_o as I and Q reference. The second approach shows that this problem simply goes away if the upper tone is used as reference in going to baseband.

The two representations also lead to different structures if the filtering is all done at baseband. Since P(t) is real, the components of $Z_1(t)$ are each obtained from the corresponding component of Z(t), using identical filters as noted above. However, if the incident signal is converted directly to baseband using the frequency $\omega_o + \nu$, the resulting signals X' and Y', are the components of Z':

$$Z'(t) = X'(t) + i\,Y'(t) \quad .$$

From these we must obtain the components of $Z_1'(t)$, according to the formulas already given. If we define

$$Z_a'(t) \equiv \int H(\sigma)\, Z'(t-\sigma)\, d\sigma \quad ,$$

then

$$Z_1'(t) = \int P_o(\sigma) Z_a'(t-\sigma) d\sigma \quad .$$

Since $P_o(t)$ is real, each component of $Z_1'(t)$ is obtained by passing the corresponding component of $Z_a'(t)$ through a filter matched to $P_o(t)$.

To obtain $Z_a'(t)$ requires four filters. In terms of the I and Q components:

$$Z_a'(t) \equiv X_a'(t) + i\,Y_a'(t)$$

$$Z'(t) \equiv X'(t) + i\,Y'(t) \, ,$$

we have

$$X_a'(t) = \int H_1(\sigma) X'(t-\sigma) d\sigma - \int H_2(\sigma) Y'(t-\sigma) d\sigma \quad ,$$

$$Y_a'(t) = \int H_2(\sigma) X'(t-\sigma) d\sigma + \int H_1(\sigma) Y'(t-\sigma) d\sigma \quad .$$

The filter impulse responses are obtained from $H(\sigma)$:

$$H_1(\sigma) = \nu P_o(\sigma) \cos(2\nu\sigma)$$

$$H_2(\sigma) = -\nu P_o(\sigma) \sin(2\nu\sigma) \quad .$$

V. SAMPLED-DATA APPROXIMATIONS TO THE MATCHED FILTER

Consider the expression

$$W(t) = \sum_n a_n Z_1(t + n\Delta)$$

for the matched filter output, in terms of the prefiltered modulation, $Z_1(t)$. This was derived first for BPSK, and also for MSK, when an appropriate choice was made for carrier frequency. Suppose this output is sampled once every δ seconds. The output samples are then

$$W_k \equiv W(k\delta) = \sum_n a_n Z_1(k\delta + n\Delta) \quad .$$

Obviously, if $\delta = \Delta/M$, for some integer M, then the output samples depend only on values of $Z_1(t)$ at multiples of δ, and the same output sequence can therefore be obtained by sampling the components of $Z_1(t)$ at the rate $1/\delta$, and then implementing the sums to form the components of the W_k by discrete delay structures. The continuous delay line is replaced by a sampled-data tapped line, such as a CCD device or digital shift register. When provided with an output formed as a weighted sum of tapped signals, the structure is a CCD or digital correlator. In this report we discuss only the approximation of matched filters by such structures, limited to the use of binary weights. Note that such a structure is a cascade of delay cells, one for each sample, and NM in number. Only every M^{th} cell is tapped to form the output.

Since only sampled outputs are available, there is no guarantee that a signal component will be sampled at the moment of peak output SNR. This is the well-known sampling loss, which we characterize by writing the SNR of sample W_k as

$$SNR(W_k) = (E_s/N_o) . L_k \quad .$$

The largest value of L_k, which represents the implementation loss of the sampled-data approximation, will be a function of "sampling phase", the interval between the actual signal arrival time and the nearest sampling time.

For either BPSK or MSK, we know that the output SNR of the matched filter is

$$SNR(t) = (E_s/N_o) \cdot \left|\frac{C(t - \tau)}{C(0)}\right|^2 ,$$

where τ is the signal arrival time, and $C(t)$ is the signal autocorrelation function. If the sampled-data receiver uses the same prefilter to form $Z_1(t)$, then

$$L_k = \left|\frac{C(k\delta - \tau}{C(0)}\right|^2$$

Suppose the nearest sampling time to the time of signal arrival is $k_o\delta$, and that

$$k_o\delta = \tau + s$$

with

$$-\frac{\delta}{2} \leq s \leq \frac{\delta}{2} .$$

Then

$$L_{k_o+j} = \left|\frac{C(j\delta + s)}{C(0)}\right|^2 ,$$

and the peak value will occur for $j = 0$:

$$L_{k_o}(s) = \left|\frac{C(s)}{C(0)}\right|^2 , \qquad |s| \leq \delta/2 .$$

If we neglect the code autocorrelation sidelobes which enter into $C(s)$ for $|s| \leq \delta/2$ (these are only the near-in sidelobes, which will be small for codes of practical interest), we obtain

$$L_{k_o}(s) = \rho_o^2(s)$$

for BPSK, and

$$L_{k_o}(s) = \rho^2(s)$$

for MSK. Obviously, sampling loss can be reduced by sampling at a high multiple of the chipping rate, but it is desirable to find other ways of controlling this loss, so that sampling rates can be as low as possible (or chipping rates as high as possible for a given sampling rate).

Another way to control sampling loss is to depart from the prefilter associated with the original matched filter receiver. This is reasonable, since the discrete delay structure approximates the continuous version of the matched filter, and it is no longer clear that the original prefilter is still optimum. Nothing can be lost, in any case, by reopening the issue of optimization, but we must now recognize that sampling phase enters as a parameter of performance.

Before discussing optimization criteria, we characterize the class of receiver structures to be considered. These represent a simple generalization in which the "prefiltered modulation", $Z_1(t)$, is defined by

$$Z_1(t) = \int G^*(\sigma)Z(\sigma + t)d\sigma ,$$

which represents the output of some filter, described by $G(\sigma)$, to the input signal. The output sequence is defined as before, by

$$W_k = \sum_n a_n Z_1(k\delta + n\Delta) ,$$

and hence the structure is appropriate to BPSK, with $G(\sigma)$ replacing $P_o(\sigma)$, and also to MSK in the S'-representation, with $G(\sigma)$ replacing $P'(\sigma)$. Since performance is characterized by SNR alone, in this analysis, we first compute output SNR for the generalized receiver, beginning with BPSK.

Expressing the input modulation as a sum of signal plus noise, we have

$$Z_1(t) = Ae^{i\beta} \sum_n a_n g(t - \tau - n\Delta) + N_1(t) ,$$

where

$$g(t) \equiv \int G^*(\sigma)P_o(\sigma + t)d\sigma$$

and

$$N_1(t) \equiv \int G^*(\sigma)N(\sigma + t)d\sigma \quad .$$

It follows that the signal component of W_k is

$$Ae^{i\beta} \sum_{\ell} C_{\ell}\, g(k\delta - \ell\Delta - \tau) \quad .$$

We neglect the contribution of code sidelobes to this term, and put $\tau = k_o\delta - s$, as before, so that the signal component of W_{k_o+j} is

$$LAe^{i\beta}\, g(j\delta + s) \quad .$$

The covariance of the noise process, $N_1(t)$, is easily computed:

$$E\, N_1^*(t)N_1(t') = 2N_o \iint G(\sigma)G^*(\sigma')\delta(\sigma + t - \sigma' - t')d\sigma\, d\sigma'$$

$$= 2N_o \int G^*(\sigma')G(\sigma' + t' - t)d\sigma' \quad ,$$

and hence the noise component of any output sample has mean squared value

$$2N_oL \int |G(\sigma)|^2\, d\sigma \quad ,$$

when code sidelobes are neglected as before. Combining these expressions yields the desired signal-to-noise ratio loss

$$L_{k_o+j} = \frac{|g(j\delta + s)|^2}{\Delta \int |G(\sigma)|^2 d\sigma}$$

Our objective now is to optimize performance by appropriately choosing the function $G(\sigma)$ which describes the prefiltering. We are free to direct the optimization to the output sample k_o (i.e. put $j=0$), since realizability can always be assured by the later addition of a delay. Thus $G(\sigma)$ will be chosen to optimize

$$L_{k_o}(s) = \frac{\left|\int G^*(\sigma)P_o(\sigma+s)d\sigma\right|^2}{\Delta\int|G(\sigma)|^2 d\sigma} \quad .$$

If the sampling phase, s, were fixed and known, then the optimum choice would be $G(\sigma) = P_o(\sigma + s)$, by an obvious application of the Schwarz Inequality. This is again the matched filter of course, with just the right delay included in the prefilter to assure that the k_o^{th} sample occurs at a time of peak output.

The simplest approach to the problem of optimizing $L_{k_o}(s)$ is to think of the sampling phase, s, as a random variable, and to replace the signal component of Z(t) by its average over sampling phase. This has the effect of replacing the factor $P_o(\sigma + s)$ by the average,

$$\overline{P}_o(\sigma) \equiv (1/\delta)\int_{-\delta/2}^{\delta/2} P_o(\sigma + s)ds \quad ,$$

in the formula for SNR. The corresponding loss factor is then

$$\frac{\left|G^*(\sigma)\overline{P}_o(\sigma)d\sigma\right|^2}{\Delta\int|G(\sigma)|^2 d\sigma} \quad .$$

This factor is maximized, again by the Schwarz Inequality, by the choice

$$G(\sigma) = \overline{P}_o(\sigma) \quad ,$$

which implies a prefilter matched to the "average pulse", $\overline{P}_o(\sigma)$. The actual loss function of a receiver using this prefilter is, of course,

$$L_{k_o}(s) = \frac{\left|\int\overline{P}_o(\sigma)P_o(\sigma+s)d\sigma\right|^2}{\Delta\int[\overline{P}_o(\sigma)]^2 d\sigma} \quad .$$

The impulse response of this filter is easily obtained:

$$G(\sigma) = (1/\delta)\int_{\sigma-\frac{\delta}{2}}^{\sigma+\frac{\delta}{2}} P_o(s)ds$$

$$= \begin{cases} 1 & ; \quad |\sigma| \leq \frac{\Delta-\delta}{2} \\ \frac{1}{\delta}\left(\frac{\Delta+\delta}{2} - |\sigma|\right) & ; \quad \frac{\Delta-\delta}{2} \leq |\sigma| \leq \frac{\Delta+\delta}{2} \\ 0 & ; \quad \frac{\Delta+\delta}{2} \leq |\sigma| \end{cases}$$

If δ is very small, i.e., many samples per chip, the portion where G (σ) varies linearly becomes very small and $G(\sigma)$ approaches $P_o(\sigma)$, leading back to the matched filter. At the other extreme, when $\delta = \Delta$, $G(\sigma)$ is a symmetrical triangular function, 2Δ wide at the base.

The loss function for this filter is easily computed, either by direct evaluation or by use of the relation

$$\int \overline{P}_o(\sigma)P_o(\sigma+s)d\sigma = \frac{\Delta}{\delta}\int_{s-\frac{\delta}{2}}^{s+\frac{\delta}{2}} \rho_o(\sigma)d\sigma \quad .$$

Strictly speaking, $L_{k_o}(s)$ was defined only for values of s in the range $|s| \leq \delta/2$, corresponding to SNR of the output sample with maximum signal. It should be obvious from our definition, however, that s can take any value and the loss $L_{k_o}(s)$ will describe the SNR of a particular output sample. For example, if $\delta/2 \leq s \leq 3\,\delta/2$,

$$L_{k_o}(s) = L_{k_o+1}(s - \sigma) \quad ,$$

which characterizes the SNR of output sample $W_{k_o} + 1$, etc.

The curves of Fig. V-1 show the variation of SNR with sampling phase for this filter, for the two cases M = 1 ($\delta = \Delta$) and M = 2 ($\delta = \Delta/2$). The curve corresponding to the original matched filter is also shown. Expanded plots of SNR loss, in dB, are presented in Fig. V-2. Explicit formulas are obtained in Appendix A-I.

A second approach to optimization is to choose $G(\sigma)$ to maximize the average SNR, averaged over sampling phase, i.e.

$$\bar{L}_{k_o} \equiv \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} L_{k_o}(s)ds \quad .$$

This is a somewhat more difficult problem, but an explicit solution can be found, and a derivation is presented in Appendix 2. The corresponding filter is almost identical to the one just derived, with almost identical performance. Average SNR losses (in dB) are given in Table I for three filters:

"A" Filter maximizing average SNR ,

"B" Filter matched to average pulse $\bar{P}_o(t)$, and

"C" Filter matched to the pulse $P_o(t)$.

TABLE I

AVERAGE SNR LOSS IN dB FOR THREE FILTERS

M	"A"	"B"	"C"
1	1.70	1.82	2.67
2	0.78	0.80	1.19
4	0.38	0.38	0.57

Worst-case losses for filters "B" and "C" are as follows:

TABLE II

WORST-CASE SNR LOSSES (IN dB)

M	"B"	"C"
1	4.26	6.02
2	1.71	2.50
4	0.78	1.16

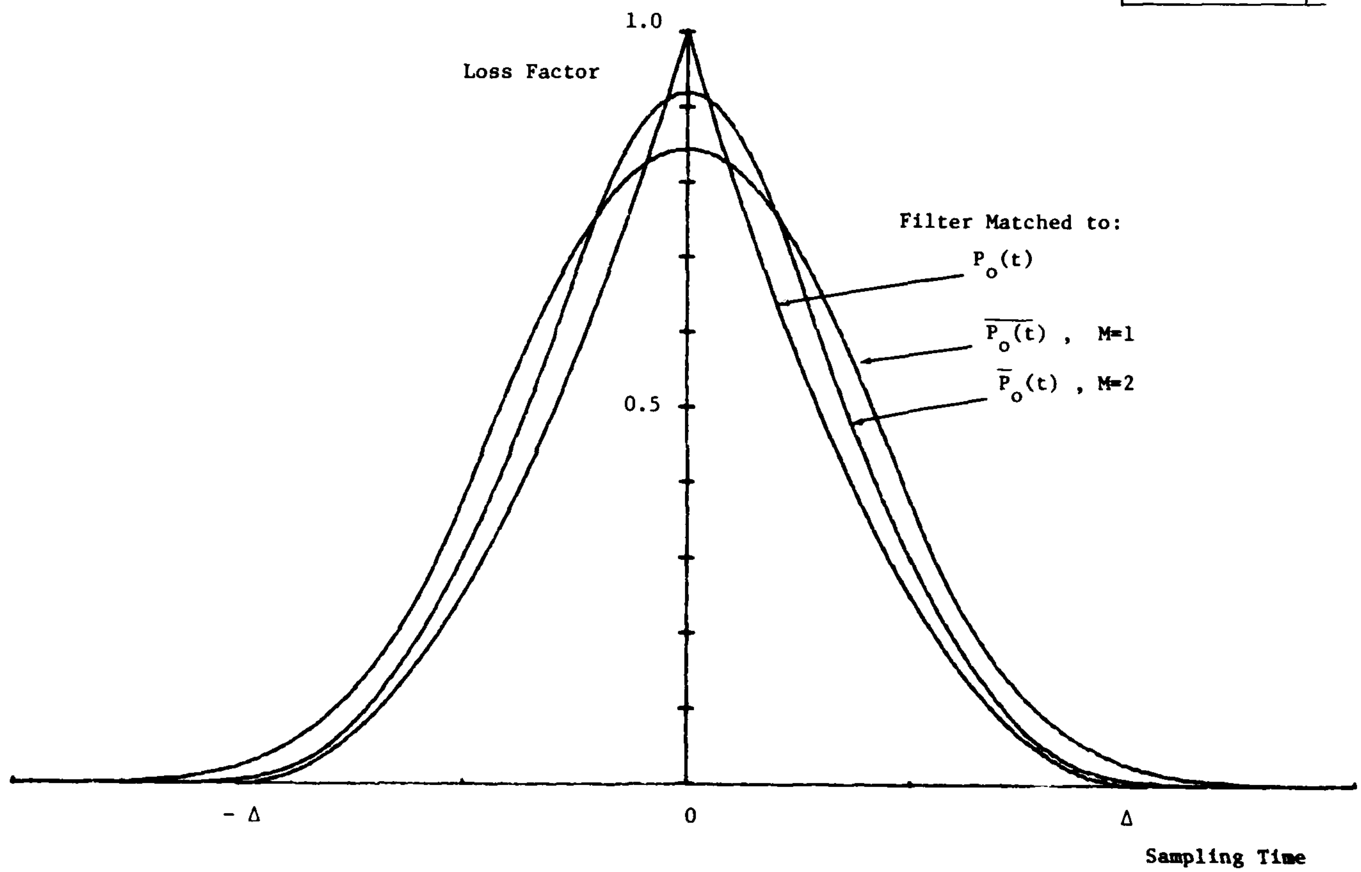

Fig. V-1. BPSK Loss Factor for three filters.

TR-550 (V-2)

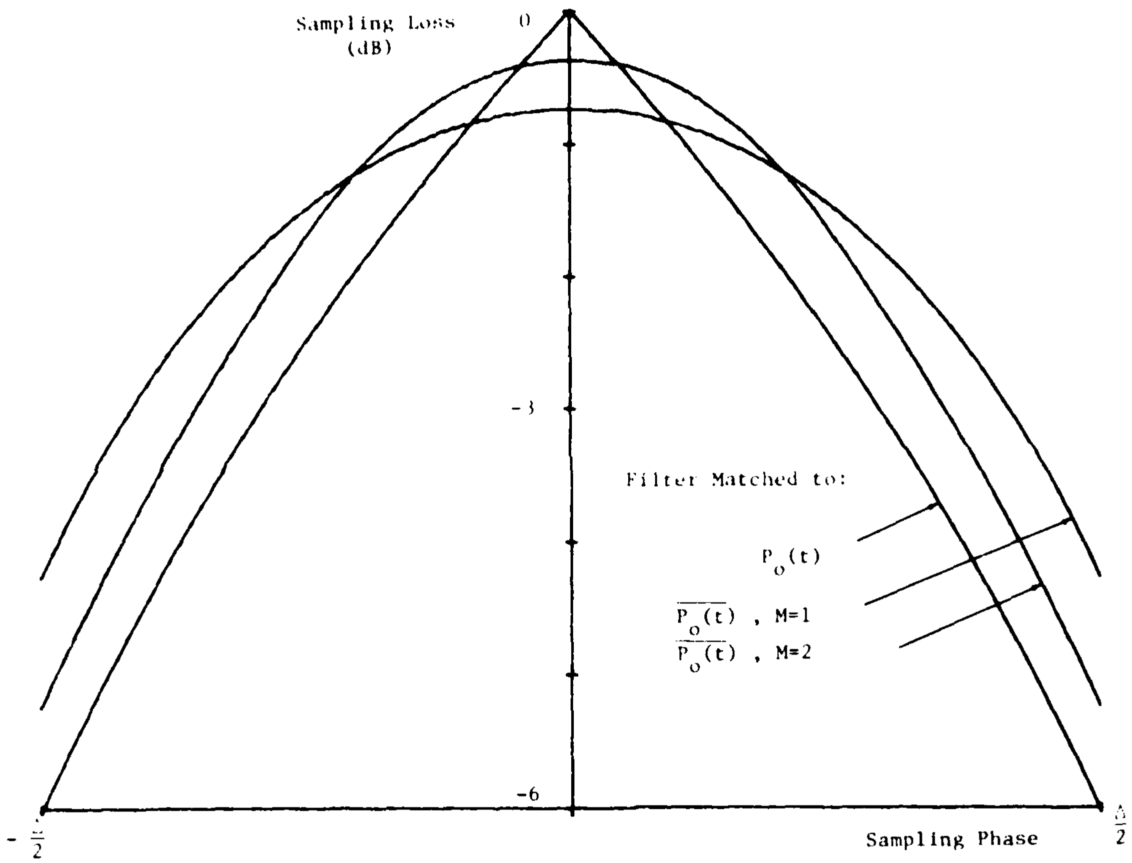

Fig. V-2. BPSK Sampling Loss in dB.

It is also possible to design a filter which yields a loss independent of sampling phase, for the one output sample having the largest SNR. The penalty paid for this uniformity is a relatively large loss, as follows:

TABLE III

SNR LOSSES FOR FILTER HAVING CONSTANT LOSS (IN dB)

M	Constant Loss
1	3.01
2	1.25
4	0.58

Derivation of this filter and its performance is presented in Appendix 3.

A broader class of sampled-data receivers emerges if every cell of the delay structure is tapped, forming an output of the form

$$W_k = \sum_{\ell} A_{\ell} \, Z_1[(k+\ell)\delta]$$

The sum would range over NM terms, corresponding to the signal duration. A sub-class results if the weights, A_{ℓ} , are equal in groups of M, and correspond to the code bits, so that

$$W_k = \sum_n a_n \sum_{m=1}^{M} Z_1(k\delta + n\Delta + m\delta)$$

This output can be written

$$W_k = \sum_n a_n \, Z_2(k\delta + n\Delta)$$

where

$$Z_2(k\delta) = \sum_{m=1}^{M} Z_1 \, [(k + m)\delta] \quad .$$

In other words, the summation over the M terms per chip can be carried out once and for all, ahead of the delav structure, since these terms always have equal weight. Moreover, the same result can be achieved ahead of the sampling step by forming

$$Z_2(t) = \sum_{m=1}^{M} Z_1(t + m\,\delta)$$

and sampling $Z_2(t)$. Finally, substituting for $Z_1(t)$, we obtain

$$Z_2(t) = \sum_{m=1}^{M} \int G^*(\sigma)\, Z(\sigma + t + m\,\delta)\,d\sigma$$

$$= \int G_2^*(\sigma)\, Z(\sigma + t)\, d\sigma$$

where $G_2(\sigma) \; \sum_{m=1}^{M} G(\sigma - m\delta)$.

In other words, the effect of the multiple, equal-weight taps can be achieved in the prefiltering, and hence this subclass offers no performance advantage, since we have already optimized the prefilter. It is interesting to note that the fil·-r we obtained by matching to the average pulse, $\overline{P}_o(\sigma)$, can be written in the form of $G_2(\sigma)$ above, namely a sum of M terms, each a displaced version of the simple triangle function:

$$G(\sigma) = \begin{cases} 1 - \frac{|\sigma|}{\delta} \; ; & |\sigma| \le \delta \\ 0 \; ; & \text{otherwise} \end{cases}$$

(exclusive of a fixed delay). Thus a receiver using a prefilter characterized by this simple triangular impulse response, in conjuction with a delay structure tapping every cell in the special way described above, would be exactly equivalent to the receiver we discussed earlier, with prefilter "B".

The same approach to sampled-data receiver optimization can be applied to MSK, beginning with the generalized prefiltered modulation

$$Z_1'(t) = \int G^*(\sigma)\, Z'(\sigma + t)\, d\sigma \; .$$

We have retained the prime in our notation to emphasize the assumption of the higher frequency, $\omega_o + \nu$, as carrier frequency. The remaining part of the processing is

$$W_k' = \sum_n a_n Z_1'(k\delta + n\Delta) \quad ,$$

obtained as before by sampling the continuous output at times $k\delta$. The calculation of output SNR is exactly parallel to the BPSK case just treated, with the resulting loss function

$$L_{k_o}(s) = \frac{\left|\int G^*(\sigma)P'(\sigma+s)d\sigma\right|^2}{\Delta\int|G(\sigma)|^2 d\sigma}$$

As before, the simplest optimization procedure is to match to the "average pulse", but we have a choice now of doing this in either of the two representations, and the results are not quite the same. Averaging in the S'-representation means choosing

$$G(\sigma) = \overline{P}'(\sigma) = \frac{1}{\delta}\int_{\sigma-\delta/2}^{\sigma+\delta/2} P'(u)du \quad ,$$

while averaging in the S-representation corresponds to the choice

$$G(\sigma) = e^{-i\nu\sigma}\,\overline{P}(\sigma)$$

$$= e^{-i\nu\sigma} \cdot \frac{1}{\delta}\int_{\sigma-\delta/2}^{\sigma+\delta/2} P(u)du \quad .$$

Although

$$P'(u) = e^{-i\nu u}\,P(u) \quad ,$$

the resulting filters are different, and it happens that the second choice (S-representation) yields better performance. The performance analysis is given in Appendix 4, and the loss curve for $M = 1$ is shown in Fig. V-3 compared to the result for the choice: $G(\sigma) = P'(\sigma)$, which is matched to the MSK pulse

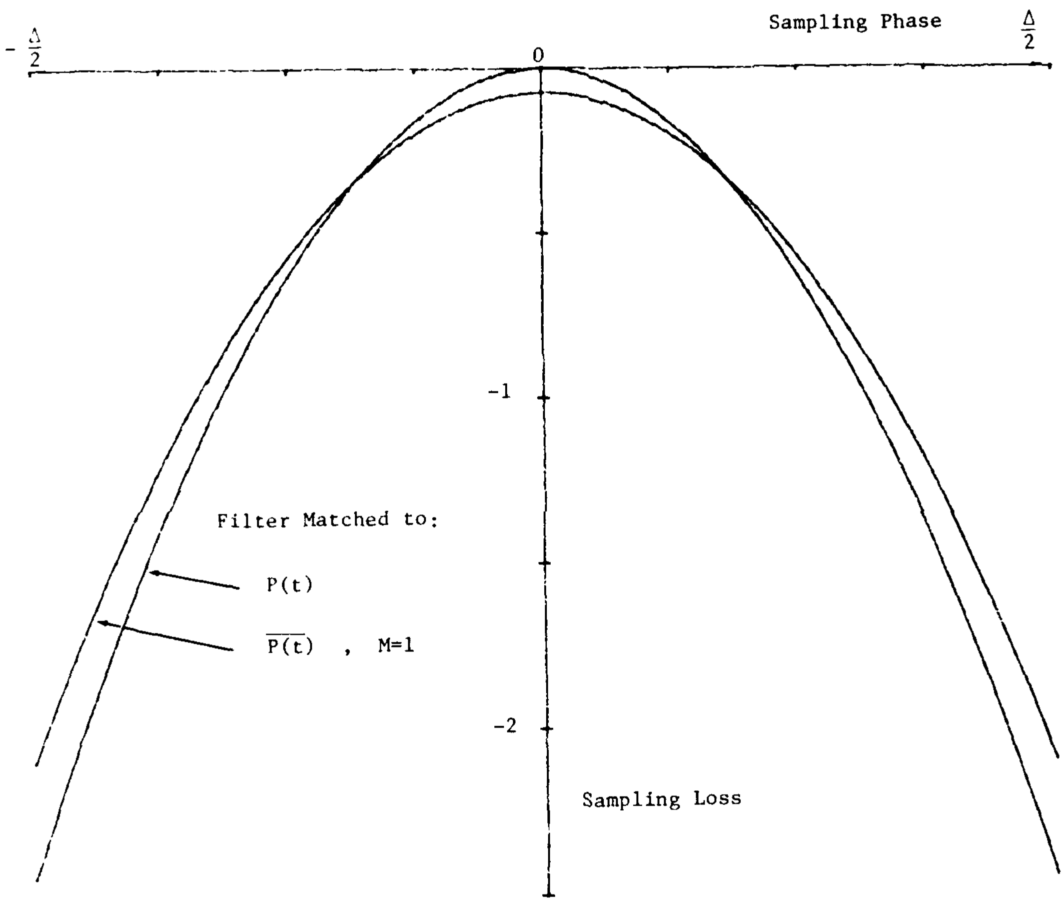

Fig. V-3. MSK Sampling loss in dB.

itself. It is seen that the results are very close, and similar curves for $M \geq 2$ show insignificant differences between matching to the MSK pulse and matching to an averaged MSK pulse. Average losses are shown in Table IV for filter "B": matched to the average pulse, and filter "C": matched to the MSK pulse itself:

TABLE IV

AVERAGE SNR LOSS FOR TWO MSK FILTERS (IN dB)

M	"B"	"C"
1	0.742	0.825
2	0.204	0.213
4	0.054	0.055

No explicit solutions have been obtained to the problems of optimizing average SNR and of obtaining a constant loss function, in the MSK case. In view of the small actual improvement of filter "B" above, relative to the conventional choice "C", there is probably little to be gained by these other approaches to optimization.

By the very nature of its autocorrelation function, $\rho(t)$, MSK shows less sensitivity to sampling phase than does BPSK. There is little to gain by filter optimization, and only about 0.5 dB average improvement of double-rate ($M = 2$) sampling over the single-rate ($M = 1$) case. This compares with a 1 dB to 1.5 dB improvement (depending on filter choice) in the BPSK average SNR, when going from single-rate to double-rate sampling. Worst-case SNR losses are also milder for MSK. The following Table compares the worst case losses of the filter "B" for BPSK (the "recommended filter", matched to an average BPSK pulse) with "C" for MSK (the conventional MSK filter, matched to the MSK pulse itself):

TABLE V

WORST-CASE SNR LOSSES (IN dB)

M	BPSK "B"	MSK "C"
1	4.26	2.44
2	1.71	0.63
4	0.78	0.16

We have found that the sampled-data MSK receiver uses the same discrete delay structure as BPSK, where each cell delays by one sample, and the structure has one tap per chip. For BPSK we showed that every cell could be tapped. with weights equal in groups of M, and the result was the same, provided a suitable change was made in the prefilter. Specifically, the receiver with prefilter $g(\sigma)$, and M taps per chip was equivalent to a receiver having one tap per chip and prefilter

$$G(\sigma) = \sum_{m=1}^{M} g(\sigma - m\delta) \qquad , \quad \delta = \Delta/M .$$

The same result holds for MSK, since the original proof depended only on the architecture of the receiver, and not on the signal modulation itself. For a given receiver tapping every sample, it is easy to find the equivalent prefilter corresponding to one tap per chip, since we start with $g(\sigma)$ and find $G(\sigma)$ by the above formula. It is not always easy to go the other way however, representing a given $G(\sigma)$ in the form required to find $g(\sigma)$. It is perhaps of interest to note that this problem can be solved for the "C" filter, since

$$G(\sigma) = P'(\sigma) = \int H(s)\, P_o(s - u)\, du.$$

We simply represent the basic pulse, P_o, as a sum of M pulses, back-to-back, each lasting one sample instead of one chip, and substitute in this integral. The result is a representation of $G(\sigma)$ in the desired form. The receiver prefilter was factured into a cascade of two filters, with impulse responses $H(s)$ and $P_o(s)$, hence it is only necessary to change the second from a filter matched to a pulse one chip in duration to a filter matched to a pulse one sample in duration. Combined with a change from one tap per chip to one tap per cell, the result is the equivalent receiver.

VI. CONCLUSIONS

The two modulations discussed in this report, BPSK and MSK, are common examples of spreading waveforms useful in a wide range of spread-spectrum systems. A great variety of signals can be designed, using these coded pulses as basic building blocks, hence the results of this study have a wide range of application. Also, it appears that sampled-data receiver structures of the kind treated here are well suited to these waveforms, where the major part of the processing gain is achieved by coherent integration of chips in a CCD or digital correlator.

One result of this study is a systematic method for the design of sampled-data receivers which can easily be extended to other spread-spectrum modulations. This method begins with the exact, continuous-time matched filter, and the discrete portion of the receiver is then inferred from the requirement that its output samples should be identical with samples of the true matched-filter output. The second step of the design process is to choose the prefilter, implemented at bandpass or base band in analog form, to optimize SNR performance as a function of sampling phase.

The other result of this analysis is the design and performance of a number of prefilters for use with BPSK and MSK. The differences in performance are not great, but the few dB they offer can in some cases be effectively traded off against some other parameter which may be near a practical bound in value. For example, one would ordinarily not use BPSK with only one sample per chip, but the fact that the 6-dB worst-case loss associated with the conventional filter can be reduced to a 3-dB loss (independent of sampling phase) might make this choice interesting in some application in which the sampling rate cannot easily be doubled. In another case, with MSK and an unconventional filter (filter "B"), double-rate sampling buys only 0.5 dB in average SNR performance, compared to single-rate sampling, a difference which could perhaps be achieved more easily by changing some link parameter, rather than doubling the clocking rate of a substantial portion of the receiver hardware.

APPENDIX 1

If we define

$$\bar{\rho}_o(s) \equiv \frac{1}{\delta} \int_{s-\frac{\delta}{2}}^{s+\frac{\delta}{2}} \rho_o(\sigma)\,d\sigma \quad ,$$

then

$$L_{k_o}(s) = \frac{\left[\bar{\rho}_o(s)\right]^2}{\frac{1}{\Delta}\int [\bar{P}_o(\sigma)]^2 d\sigma}$$

One immediately finds that

$$\frac{1}{\Delta}\int [\bar{P}_o(\sigma)]^2 d\sigma = 1 - \frac{1}{3M}$$

where $\qquad M = \Delta/\delta \quad .$

Since $\bar{\rho}_o(-s) = \bar{\rho}_o(s)$, we evaluate $\bar{\rho}_o(s)$ for positive s. When $0 \leq s \leq \frac{\delta}{2}$, we have

$$\bar{\rho}_o(s) = \frac{1}{\delta}\int_o^{s+\frac{\delta}{2}} (1 - \frac{\sigma}{\Delta})\,d\sigma + \frac{1}{\delta}\int_o^{\frac{\delta}{2}-s} (1 - \frac{\sigma}{\Delta})\,d\sigma$$

$$= \frac{1}{\delta}\left[\delta - \frac{1}{2\Delta} \quad s^2 + \frac{\delta^2}{2})\right]$$

$$= 1 - \frac{1}{4M} - M\,\frac{s^2}{\Delta^2} \quad .$$

When $\qquad \frac{\delta}{2} \leq s \leq \Delta - \frac{\delta}{2}$, we have

$$\bar{\rho}_o(s) = \frac{1}{\delta}\int_{s-\frac{\delta}{2}}^{s+\frac{\delta}{2}} (1 - \frac{\sigma}{\Delta})d\sigma = 1 - \frac{s}{\Delta} \quad ,$$

and when $\Delta - \frac{\delta}{2} \leq s \leq \Delta + \frac{\delta}{2}$.

$$\bar{\rho}_o(s) = \frac{1}{\delta}\int_{s-\frac{\delta}{2}}^{\Delta} (1 - \frac{\sigma}{\Delta})d\sigma$$

$$= \frac{1}{\delta}\left[\Delta + \frac{\delta}{2} - s - \frac{1}{2\Delta}(\Delta^2 - s^2 + s\delta - \frac{\delta^2}{4})\right]$$

$$= \frac{M}{2}\left(\frac{s}{\Delta} - 1 - \frac{1}{2M}\right)^2 \quad .$$

For larger s, $\bar{\rho}_o(s)$ vanishes. Collecting the formulas:

$$\bar{\rho}_o(s) = \begin{cases} 1 - \frac{1}{4M} - M(\frac{s}{\Delta})^2 & ; \quad 0 \leq |\frac{s}{\Delta}| \leq \frac{1}{2M} \\ 1 - \frac{|s|}{\Delta} & ; \quad \frac{1}{2M} \leq \frac{|s|}{\Delta} \leq 1 - \frac{1}{2M} \\ \frac{M}{2}\left(1 + \frac{1}{2M} - \frac{|s|}{\Delta}\right)^2 & ; \quad 1 - \frac{1}{2M} \leq \frac{|s|}{\Delta} \leq 1 + \frac{1}{2M} \\ 0 & ; \quad \frac{|s|}{\Delta} \geq 1 + \frac{1}{2M} \end{cases} ,$$

and

$$L_{k_o}(s) = \frac{\left[\bar{\rho}_o(s)\right]^2}{1 - \frac{1}{3M}} \quad .$$

APPENDIX 2

For an arbitrary filter, $G(\sigma)$, the SNR as a function of sampling phase is proportional to the ratio

$$\frac{\left|\int G(\sigma)\, P_o(\sigma + s) d\sigma\right|^2}{\int |G(\sigma)|^2 d\sigma}$$

We now wish to choose $G(\sigma)$ in order to maximize the average value of this expression, treating sampling phase as a uniformly distributed random variable. The solution to this problem will be sketched briefly here. The numerator is

$$\iint G^*(\sigma)G(\sigma')P_o(\sigma + s)P_o(\sigma' + s)d\sigma\, d\sigma' \quad ,$$

and its average can be expressed as

$$\iint G^*(\sigma)G(\sigma')A(\sigma,\sigma')d\sigma\, d\sigma'$$

where

$$A(\sigma,\sigma') \equiv \frac{1}{\delta}\int_{-\delta/2}^{\delta/2} P_o(\sigma + s)P_o(\sigma' + s)\, ds \quad .$$

The problem of maximizing the ratio

$$\frac{\iint G^*(\sigma)G(\sigma')A(\sigma,\sigma')d\sigma\, d\sigma'}{\int |G(\sigma)|^2 d\sigma} \quad .$$

is a standard one, whose solution is to take $G(\sigma)$ proportional to that eigenfunction of the operator A, corresponding to the largest eigenvalue of A. The eigenfunctions of A are defined by the equation

$$\int A(\sigma,\sigma')f(\sigma')d\sigma' = \lambda f(\sigma) \quad .$$

Together, they span a subspace of functions, and it is clearly disadvantageous to choose a $G(\sigma)$ which has a component outside this subspace, since this component increases the denominator of our ratio without helping the numerator. This heuristic argument is easily made rigorous, and once confined to the subspace in question, $G(\sigma)$ can be expanded in eigenfunctions:

$$G(\sigma) = \sum_m \beta_m \psi_m(\sigma) \quad .$$

The positive definite operator A will turn out to have a discrete spectrum, and we will have

$$\iint G^*(\sigma)A(\sigma,\sigma')G(\sigma')d\sigma\, d\sigma' = \sum_m \lambda_m |\beta_m|^2 \quad ,$$

while

$$\int |G(\sigma)|^2 d\sigma = \sum_m |\beta_m|^2$$

(we have assumed the ψ_m to be orthogonal and normalized). If λ_o is the largest eigenvalue (they are positive), then the choice $G(\sigma) = f_o(\sigma)$ solves our problem.

Since $P_o(t)$ is a simple pulse, it is easy to evaluate $A(\sigma,\sigma')$ explictly, although it is complicated to write down. As a function of σ, $A(\sigma, \sigma')$ is constant when $|\sigma| < \frac{1}{2}(\Delta - \delta)$ and zero when $|\sigma| > \frac{1}{2}(\Delta + \delta)$. Between these bounds, $A(\sigma,\sigma')$ is either zero, or linear (increasing or decreasing) in σ. It follows that the eigenfunctions are zero when $|\sigma| > \frac{1}{2}(\Delta + \delta)$ and constant when $|\sigma| < \frac{1}{2}(\Delta - \delta)$. Only the portion of the eigenfunctions in the remaining range needs to be determined, and by differentiating the integral equation twice, a differential equation results. This equation is simply the equation of a sinusoid, and by matching boundary conditions, a discrete spectrum is obtained. The solution with maximum eigenvalue joins the constant portion to the value zero at the ends of the range with a portion of a sinusoid that differs only very slightly from a straight line, the corresponding solution, matched to the averaged pulse. The SNR attained is determined by the eigenvalue itself, whose evaluation leads to the values given in the text.

APPENDIX 3

To obtain a SNR independent of sampling phase, it is necessary to choose $G(\sigma)$ so that

$$\int G(\sigma)\, P_o(\sigma + s)\, d\sigma$$

$$= \int_{-\frac{\Delta}{2}+s}^{\frac{\Delta}{2}+s} G(\sigma)\, d\sigma$$

is independent of s, for $|s| \leq \delta/2$. By differentiating this expression, it is seen that we require

$$G(\frac{\Delta}{2} + s) = G(-\frac{\Delta}{2} + s) \quad , \quad -\frac{\delta}{2} \leq s \leq \frac{\delta}{2} \quad .$$

Thus, we can put

$$G(\sigma) = a(\sigma) \text{ for } |\sigma| \leq \frac{\Delta - \delta}{2}$$

and

$$G(\frac{\Delta}{2} + \sigma) = G(-\frac{\Delta}{2} + \sigma) = b(\sigma) \quad , \quad \text{for } |\sigma| \leq \delta/2 \quad ,$$

where $a(\sigma)$ and $b(\sigma)$ are arbitrary, and $G(\sigma)$ is taken equal to zero elsewhere. Since SNR is now independent of s, we can choose $s = -\delta/2$, and the loss factor will be

$$\left[\int_{-\frac{\Delta-\delta}{2}}^{+\frac{\Delta-\delta}{2}} a(\sigma)\, d\sigma + \int_{-\delta/2}^{\delta/2} b(\sigma)\, d\sigma \right]^2$$

$$\int_{-\frac{\Delta - \delta}{2}}^{+\frac{\Delta - \delta}{2}} a^2(\sigma)\,d\sigma \;+\; 2\int_{-\delta/2}^{\delta/2} b^2(\sigma)\,d\sigma \quad .$$

It is not hard to show, by means of the calculus of variations, that this ratio is maximized when $a(\sigma)$ and $b(\sigma)$ are constant, with $b = a/2$. This is the filter referred to in the text, and the loss factor resulting is given by

$$L = 1 - \frac{\delta}{2\Delta} = 1 - \frac{1}{2M}$$

APPENDIX 4

The basic expression for the loss factor can be written in the form

$$L_M(s) = \frac{|B_M(s)|^2}{D_M}$$

where

$$B_M(s) \equiv \frac{1}{\Delta}\int G_M^*(\sigma)P(\sigma + s)d\sigma$$

$$D_M \equiv \frac{1}{\Delta}\int |G_M(\sigma)|^2 d\sigma$$

and

$$G_M(\sigma) = \overline{P}(\sigma) = \frac{1}{\delta}\int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} P(\sigma + u)du$$

The MSK autocorrelation function is

$$\rho(t) \equiv \frac{1}{\Delta}\int P(\sigma)P(\sigma + t)d\sigma$$

$$= \begin{cases} (1 - \frac{|t|}{2\Delta}) \cos (\frac{\pi t}{2\Delta}) + \frac{1}{\pi} \sin (\frac{\pi |t|}{2\Delta} \ ; & |t| \leq 2\Delta \\ 0 \ ; & \text{otherwise} \end{cases} .$$

In terms of $\rho(t)$, we have

$$B_M(s) = \frac{1}{\Delta}\int \left\{ \frac{1}{\delta} \int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} P(\sigma + u)du \right\} P(\sigma + s)d\sigma$$

$$= \frac{1}{\delta}\int_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \rho(s-u)du = \frac{1}{\delta}\int_{s - \frac{\delta}{2}}^{s + \frac{\delta}{2}} \rho(u)du \quad ,$$

and also

$$D_M = \frac{1}{\Delta}\int\left\{\frac{1}{\delta^2}\iint_{-\frac{\delta}{2}}^{\frac{\delta}{2}} P(\sigma + u)P(\sigma + u')dudu'\right\} d\sigma$$

$$= \frac{1}{\delta^2}\iint_{-\frac{\delta}{2}}^{\frac{\delta}{2}} \rho(u' - u)dudu' = \frac{2}{\delta^2}\int_0^{\delta} \rho(w)(\delta - w)dw \quad .$$

In this last integral we put $w = x\sigma$ and substitute for $\rho(w)$, with the result

$$D_M = 2\int_0^1 \left\{(1 - \frac{x}{2M})\cos(\frac{\pi x}{2M}) + \frac{1}{\pi}\sin(\frac{\pi x}{2M})\right\}(1 - x)dx,$$

where $M = \Delta/\delta$. Direct evaluation yields

$$D_M = \frac{4M}{\pi^2}\left\{2(M + 1)-(2M-1)\cos(\frac{\pi}{2M})-\frac{6M}{\pi}\sin(\frac{\pi}{2M})\right\}$$

For $M = 1$, this simplifies to

$$D_1 = \frac{8}{\pi^2}(2-\frac{3}{\pi})$$

Turning to $B_M(s)$, we note that it is an even function of s and that

$$F(u) \equiv \frac{1}{\delta}\int\left\{(1-\frac{u'}{2\Delta})\cos(\frac{\pi u'}{2\Delta})+\frac{1}{\pi}\sin(\frac{\pi u'}{2\Delta})\right\}du'$$

can be evaluated as

$$F(u) = \frac{1}{\pi}(2M - \frac{u}{\delta}) \sin(\frac{\pi u}{2M\delta}) - \frac{4M}{\pi^2} \cos(\frac{\pi u}{2M\delta}) \quad .$$

When $s \geq 2\Delta + \frac{\delta}{2}$, we have $B_M(s) = 0$, since the averaging interval is beyond the range of $\rho(u)$. When $2\Delta - \frac{\delta}{2} \leq s \leq 2\Delta + \frac{\delta}{2}$, we have

$$B_M(s) = F(2\Delta) - F(s - \frac{\delta}{2})$$

$$= \frac{4M}{\pi^2} + \sin(\frac{\pi s}{2\Delta}) \left\{ \frac{4M}{\pi^2} \sin(\frac{\pi}{4M}) - \frac{1}{\pi}(2M + \frac{1}{2} - \frac{MS}{\Delta})\cos(\frac{\pi}{4M}) \right\}$$

$$+ \cos(\frac{\pi s}{2\Delta}) \left\{ \frac{4M}{\pi^2} \cos(\frac{\pi}{4M}) + \frac{1}{\pi}(2M + \frac{1}{2} - \frac{MS}{\Delta})\sin(\frac{\pi}{4M}) \right\}$$

Next, when $\frac{\delta}{2} \leq s \leq 2\Delta - \frac{\delta}{2}$,

$$B_M(s) = F(s + \frac{\delta}{2}) - F(s - \frac{\delta}{2})$$

$$= \sin(\frac{\pi s}{2\Delta}) \left\{ \frac{8M}{\pi^2} \sin(\frac{\pi}{4M}) - \frac{1}{\pi} \cos(\frac{\pi}{4M}) \right\}$$

$$+ \frac{2M}{\pi} \cos(\frac{\pi s}{2\Delta})(2 - \frac{s}{\Delta})\sin(\frac{\pi}{4M}) \quad ,$$

Finally, when $0 \leq s \leq \frac{\delta}{2}$, we obtain

$$B_M(s) = F(s + \frac{\delta}{2}) + F(\frac{\delta}{2} - s) - 2F(0)$$

$$= \frac{8M}{\pi^2} - \frac{2Ms}{\pi\Delta} \cos(\frac{\pi}{4M})\sin(\frac{\pi s}{2\Delta})$$

$$+ \left\{ \frac{1}{\pi}(4M - 1)\sin(\frac{\pi}{4M}) - \frac{8M}{\pi^2} \cos(\frac{\pi}{4M}) \right\} \cos(\frac{\pi s}{2\Delta})$$

The peak SNR always occurs in the interval

$$-\frac{\Delta}{2} \leq s \leq +\frac{\Delta}{2} ,$$

in which case the explicit loss factor is

$$L_M(s) = \frac{\left[\frac{8M}{\pi} - \frac{2Ms}{\Delta} \sin\left(\frac{\pi s}{2\Delta}\right)\cos\left(\frac{\pi}{4M}\right) + \left[(4M-1)\sin\left(\frac{\pi}{4M}\right) - \frac{8M}{\pi} \cos\left(\frac{\pi}{4M}\right)\right] \cos\left(\frac{\pi s}{2\Delta}\right)\right]^2}{4M \left\{2(M+1) - (2M-1)\cos\left(\frac{\pi}{2M}\right) - \frac{6M}{\pi}\sin\left(\frac{\pi}{2M}\right)\right\}} .$$

For $M = 1$, this reduces to

$$L_1(s) = \frac{2}{\pi} \frac{\left[2 + \left(\frac{3\pi}{8} - 1\right)\sqrt{2}\cos\left(\frac{\pi s}{2\Delta}\right) - \frac{1}{\sqrt{2}} \left(\frac{\pi s}{2\Delta}\right) \sin\left(\frac{\pi s}{2\Delta}\right)\right]^2}{2\pi - 3} ,$$

which is shown in Fig. V - 3.

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Certain spread-spectrum systems make use of waveforms built of basic pulses spread by PN modulation of some kind. In this study the two most common modulation waveforms, BPSK and MSK are discussed in detail. The objective is to derive receiver structures utilizing sampled-data integration of the chips, for example, using CCD or digital correlators. A constructive design procedure is described, together with an analysis of performance in terms of SNR loss due to sampling. It is shown that sampling loss can be reduced by proper choice of the prefilter which precedes the sampling of the data.

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